Lösungen zum Übungsblatt 2

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Problem 1: Random Number Generation

Introduction: Methods for computing homogeneous distributed random numbers are offered in almost every programming language. To generate numbers that are distibuted in an other way, e.g. gaussian or possion distibuted, it is necessary to use transformation methods. This paper describes the way to get a stochastic variable with the exponential distribution $\rho_Y(y) = \exp(-y)$ on the half-line $[0, \infty[$ and its implementation on a computer.

Methods: By using the fundamental transformation law of probabilities and the fact, that the exponential function is invertable, we derive a relation between the homogenious distributed stochastic variable X and the wanted Y with exponential distribution.

Results: By putting a homogenous distributed x ($x \in [0,1]$) in $y(x) = -\ln(x)$ we get a $y \in [0, \infty[$ that is exponentially distributed given by $\rho_Y(y) = \exp(-y)$. Therefore, to produce exponetially distributed numbers, we need to use the build-in methods of the programming language to get homogenous distributed randum numbers x between 0 and 1 and then calculate $-\ln(x)$.

Discussion: The described method can be used to simulate physical processes that follow exponential distributions, e.g. radioactive decay.

Appendix As discussed in the lecture, the fundamental transformation law of probabilities is given by

$$\rho_Y(y) = \rho_X(x) \left| \frac{dx}{dy} \right| \quad . \tag{1}$$

With $\rho_X(x) = 1$ and $\rho_Y(y) = \exp(-y)$, we obtain:

$$\exp(-y) = \left| \frac{dx}{dy} \right| \quad , \tag{2}$$

what leads after one integration to:

$$x(y) = \pm \exp(-y) \quad . \tag{3}$$

The negative case can be ignored because of $x \in [0,1]$ and we finally get:

$$x(y) = \exp(-y) \quad \Rightarrow \quad y = -\ln(x) \quad . \tag{4}$$